



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## SOLUTION BY THE PROPOSER.

As this triangle is designed to show the slope of earth embankments the word 'slope' is defined as it is understood by the civil engineer: the ratio of horizontal to vertical distance.

Let  $AD$ ,  $BE$ ,  $CF$  be the medians of the required triangle,  $ABC$ , then

$$(1) \quad 2 \cot ADB = \cot C - \cot B,$$

$$(2) \quad 2 \cot BEC = \cot A - \cot C,$$

$$(3) \quad 2 \cot CFA = \cot B - \cot A,$$

and hence  $\cot ADB + \cot BEC + \cot CFA = 0$ . If we take  $\cot ADB = \frac{2}{3}$  then from the assigned values: (a)  $\cot BEC = -1$ ,  $\cot CFA = \frac{1}{3}$ ; (b)  $\cot BEC = \frac{1}{3}$ ,  $\cot CFA = -1$ . Considering case (a), these values inserted in (1), (2), (3) give two independent equations, which are to be combined with the identity,

$$(4) \quad \cot B \cot C + \cot C \cot A + \cot A \cot B = 1.$$

Putting, from (2) and (3),  $\cot B = \cot A + \frac{2}{3}$ ,  $\cot C = \cot A + 2$ , equation (4) becomes

$$9 \cot^2 A + 16 \cot A + 1 = 0;$$

whence,

$$\cot A = \frac{-8 \pm \sqrt{55}}{9}; \quad \cot B = \frac{-2 \pm \sqrt{55}}{9}; \quad \text{and} \quad \cot C = \frac{10 \pm \sqrt{55}}{9}.$$

Since a triangle can have only one obtuse angle only one cotangent can be negative. Consequently, the  $(-)$  before the radical does not lead to a solution. Using the  $(+)$  sign,

$$A = 93^\circ 42' 42'', \quad B = 58^\circ 57' 37'', \quad C = 27^\circ 19' 41''.$$

When  $c = 10.00$ ,  $a = 21.74$  and  $b = 18.66$ .

Case (b) can be treated in the same way and we get

$$\cot A = \frac{8 + \sqrt{55}}{9}, \quad \cot B = \frac{-10 + \sqrt{55}}{9}, \quad \cot C = \frac{2 + \sqrt{55}}{9}.$$

## 2745 [1919, 37]. Proposed by G. I. HOPKINS, Manchester, N. H.

A recent English publication contains the following method of constructing a regular polygon of 17 sides: Draw the radius  $CB$  perpendicular to the diameter  $AQ$  of the circle whose center is  $B$ . On  $BC$  lay off  $BD$  equal to one-fourth of  $BC$ . On  $BA$ , lay off  $BE$  and draw  $DE$  making angle  $BDE$  one fourth of angle  $BDA$ . On  $BQ$  lay off  $BF$  and draw  $DF$ , making angle  $FDE$   $45^\circ$ . On  $AF$  as diameter, construct semi-circle  $FHA$  intersecting  $CB$  in  $H$ . With  $E$  as center and  $EH$  as radius construct semi-circle  $LHK$  intersecting  $CB$  in  $L$ . At the points  $L$  and  $K$  draw the ordinates  $NL$  and  $MK$ . Bisect the arc  $NM$  and let  $P$  be the point of bisection. Then the chord  $NP(=MP)$  is a side of the regular polygon of 17 sides. Is the method of construction correct?

## I. SOLUTION BY C. H. CHEPMELL, Hove, England.

The abscissas  $BK$ ,  $BL$ , and their ordinates to  $M$  and  $N$ , make the angles  $MBA$ ,  $NBA$  equal to  $10\pi/17$ , and  $6\pi/17$  respectively. Consequently the difference of these two angles, the angle  $MBN$ , is equal to  $4\pi/17$ . This justifies the claim of the construction.

But tables of all the trigonometrical functions of  $n\pi/17$  are not readily available; and it may be more satisfactory if we outline the connection between the numerical value of  $NM$  and the numerical value of some function given in a standard work.

Taking the radius of the circle  $ACQ$  as unity, we find

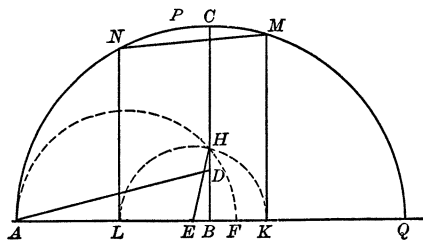
$$BE = \frac{-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}}{16},$$

$$BF = \frac{+1 - \sqrt{17} + \sqrt{34 - 2\sqrt{17}}}{16},$$

$$EH = \frac{\sqrt{2(\sqrt{17} - 3)(2\sqrt{17} + \sqrt{34 + 2\sqrt{17}})}}{16} = \frac{\beta}{16},$$

$$AK = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} + \beta}{16},$$

$$AL = \frac{+17 + \sqrt{17} - \sqrt{34 + 2\sqrt{17}} - \beta}{16},$$



$$\begin{aligned}
KQ &= \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} - \beta}{16}, \\
LQ &= \frac{+15 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}} + \beta}{16}, \\
LN &= \sqrt{AL \cdot LQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} - 2\beta'}, \\
KM &= \sqrt{AK \cdot KQ} = \frac{1}{8} \sqrt{+34 + 2\sqrt{17} + 2\sqrt{34 + 2\sqrt{17}} + 2\beta'}, \\
[\beta' &= \sqrt{2(\sqrt{17} - 3)(2\sqrt{17} - \sqrt{34 + 2\sqrt{17}})}; \quad \text{and} \quad \beta \times (-1 - \sqrt{17} + \sqrt{34 + 2\sqrt{17}}) = 4 \cdot \beta'], \\
LN \times KM &= \frac{1}{64} \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} + \sqrt{34 - 2\sqrt{17}})} = \frac{4\alpha}{64}, \\
NM^2 &= LK^2 + (KM - LN)^2 \\
&= 4 \cdot EH^2 + KM^2 + LN^2 - \frac{8\alpha}{64} \\
&= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 8\alpha}{64}, \\
[\alpha' &= \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}; \quad \text{and} \quad 4\alpha = \alpha' \times (-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})], \\
\therefore NM^2 &= \frac{+136 - 8\sqrt{17} + 8\sqrt{34 - 2\sqrt{17}} - 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64}.
\end{aligned}$$

And, in the circle with radius equal to unity,  $NM$  represents the value  $2 \sin (2\pi/17) \times 1$ ; and therefore

$$\begin{aligned}
4 \cos^2 \frac{2\pi}{17} &= 4 - NM^2 \\
&= \frac{+120 + 8\sqrt{17} - 8\sqrt{34 - 2\sqrt{17}} + 2(-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}})\alpha'}{64} \\
&= \left[ \frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \alpha'}{8} \right]^2
\end{aligned}$$

and

$$2 \cos \frac{2\pi}{17} = \frac{-1 + \sqrt{17} + \sqrt{34 - 2\sqrt{17}} + \sqrt{2(\sqrt{17} + 3)(2\sqrt{17} - \sqrt{34 - 2\sqrt{17}})}}{8}.$$

And this value of  $2 \cos (2\pi/17)$  will be found to agree with that given in Klein's *Famous Problems in Elementary Geometry* (Beman & Smith), though our  $\alpha'$  is there written in a different form.

## II. HISTORICAL NOTE BY R. C. ARCHIBALD, Brown University.

This method of construction is due to H. W. Richmond, *Quarterly Journal of Mathematics*, Volume 26, 1893, pp. 206-207; and *Mathematische Annalen*, Volume 67, 1909, pp. 460-461. It is reproduced on page 34 of H. P. Hudson's *Ruler and Compasses*, London, 1916.

Various constructions of the regular polygon of seventeen sides were reviewed by R. Goldenring in his *Die elementargeometrischen Konstruktionen des regelmässigen Siebzehnecks* (Leipzig, 1915), but many omissions in this professedly complete survey were noted by the writer in the *Bulletin of the American Mathematical Society*, vol. 22, 239-246. The first solution in an English publication, given by Lowry in 1819,<sup>1</sup> was reproduced in this *Monthly*<sup>2</sup> in 1899 and 1914. Other solutions and historical notes are set forth in the articles printed above, pages 322-326.

### 2767 [1919, 171]. Proposed by W. W. JOHNSON, U. S. Naval Academy.

Let the complex quantities  $p$ ,  $q$ , and  $r$  satisfy the relation  $p^2 + q^2 + r^2 = 0$ ; prove that the corresponding vectors  $OP$ ,  $OQ$ , and  $OR$  are such that if any two of them are taken as conjugate semi-diameters of an ellipse, the third lies on the minor axis, and its length is the distance from the center to either focus.

### SOLUTION BY A. PELLETIER, Montreal, Can.

Let  $(x^2/a^2) + (y^2/b^2) = 1$ , be the equation of the ellipse having  $OP$  and  $OQ$  for conjugate semi-diameters ( $2a$  and  $2b$  being the axes, and  $a \geq b$ ). If  $\alpha$ ,  $\alpha'$ ,  $\alpha''$  are the respective arguments of

<sup>1</sup> *The Mathematical Repository*, new series, vol. 4, p. 160; Lowry's proof occupies pages 160-168.

<sup>2</sup> Volume 6, p. 239 and volume 21, p. 252.